SAMPLE PAPER

issued by CBSE for Term I Exams (2021-22)
Mathematics (041) - Class 12

Time Allowed : 90 Minutes **Max. Marks : 40**

General Instructions :

1. This question paper contains three Sections - **Section A, B and C**. Each section is compulsory.
2. **Section A** carries **20 Questions** and you need to attempt any **16 Questions**.
   Section B carries **20 Questions** and you need to attempt any **16 Questions**.
   Section C carries **10 Questions** and you need to attempt any **8 Questions**.
3. There is **no negative marking**. All questions carry equal marks.

Section A

Questions in this section carry 1 mark each.

In this section, attempt **any 16 questions** (from **01 - 20**).

01. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) -1 (d) 1

Sol. $\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]=\sin\frac{\pi}{2}=1$.

02. The value of k, $(k<0)$ for which the function f defined as $f(x)=\begin{cases}\frac{1-\cos kx}{x\sin x}, & x\neq 0\\ \frac{1}{2}, & x=0\end{cases}$ is continuous at $x=0$ is

(a) ± 1 (b) -1

(c) $\pm\frac{1}{2}$ (d) $\frac{1}{2}$

Sol. As f is continuous at $x=0$ so, $\lim_{x\to 0}f(x)=f(0)$ i.e., $\lim_{x\to 0}\frac{1-\cos kx}{x\sin x}=\frac{1}{2}$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin^2\frac{kx}{2}}{x^2\left(\frac{\sin x}{x}\right)}=\frac{1}{2}$$

$$\Rightarrow 2\lim_{(kx/2)\to 0}\frac{\sin^2\frac{kx}{2}}{\frac{k^2x^2}{4}}\times\frac{k^2}{4}\times\lim_{x\to 0}\frac{1}{\left(\frac{\sin x}{x}\right)}=\frac{1}{2}$$

$\Rightarrow 2(1)^2 \times \frac{k^2}{4} \times \frac{1}{1} = \frac{1}{2}$

$\Rightarrow k^2 = 1 \qquad \Rightarrow k = \pm 1$

But $k < 0$ so, $k = -1$.

03. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{ when } i \neq j \\ 0, \text{ when } i = j \end{cases}$, then A^2 is

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \Rightarrow A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A^2 = AA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

04. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is

(a) 4 (b) –4

(c) ±4 (d) 0

Sol. As $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix so, $|A| = \begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix} = 0$

$\Rightarrow 2k^2 - 32 = 0$

$\Rightarrow k^2 = 16$

$\therefore k = \pm 4$.

05. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing

(a) $(-\infty, 2) \cup (2, \infty)$ (b) $(2, \infty)$

(c) $(-\infty, 2)$ (d) $(-\infty, 2] \cup (2, \infty)$

Sol. $\because f'(x) = 2x - 4$

For $f'(x) = 2x - 4 = 0 \qquad \Rightarrow x = 2$

Note that $f'(x) > 0$ for all $x \in (2, \infty)$.

Therefore, $f(x)$ is strictly increasing in $x \in (2, \infty)$.

06. Given that A is a square matrix of order 3 and $|A| = -4$, then $|adj.A|$ is equal to

(a) –4 (b) 4

(c) –16 (d) 16

Sol. $\because |adj.A| = |A|^{n-1}$, where n is order of A

$\therefore |adj.A| = (-4)^{3-1} = 16$.

07. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1,1), (1,2), (2,2), (3,3)\}$.
Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

(a) (1, 1) (b) (1, 2)

(c) (2, 2) (d) (3, 3)

Sol. Note that the presence of (1, 2) in R is disturbing symmetry of the relation.
So, we should remove the ordered pair (1, 2) so that R becomes a symmetric relation and hence, equivalence relation as well.

08. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of $a+b-c+2d$ is

(a) 8 (b) 10
(c) 4 (d) –8

Sol. By def. of equality of matrices, we get $2a+b=4,\ a-2b=-3,\ 5c-d=11,\ 4c+3d=24$.
On solving the equations, we get $a=1,\ b=2,\ c=3,\ d=4$.
Hence, $a+b-c+2d=1+2-3+2(4)=8$.

09. The point at which the normal to the curve $y=x+\frac{1}{x},\ x>0$ is perpendicular to the line $3x-4y-7=0$ is

(a) (2, 5/2) (b) (±2, 5/2)
(c) (−1/2, 5/2) (d) (1/2, 5/2)

Sol. Here $\frac{dy}{dx}=1-\frac{1}{x^2}$ so, slope of normal to the curve $y=x+\frac{1}{x}$ will be $\frac{-x^2}{x^2-1}$.
Also the normal is perpendicular to $3x-4y-7=0$ so, $\left(\frac{-x^2}{x^2-1}\right)\left(\frac{3}{4}\right)=-1$

$\Rightarrow 3x^2=4x^2-4 \quad \Rightarrow x^2=4 \quad \therefore x=2,\ -2$
But $x>0$ so, $x=2$.

Putting $x=2$ in $y=x+\frac{1}{x}$, we get : $y=2+\frac{1}{2}=\frac{5}{2}$.

Therefore, the required point is $\left(2,\frac{5}{2}\right)$.

10. $\sin(\tan^{-1}x)$, where $|x|<1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$
(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

Sol. Let $\tan^{-1}x=\theta \Rightarrow x=\tan\theta$.
So, $\sin(\tan^{-1}x)=\sin\theta=\frac{1}{\operatorname{cosec}\theta}=\frac{1}{\sqrt{1+\cot^2\theta}}=\frac{\tan\theta}{\sqrt{\tan^2\theta+1}}=\frac{x}{\sqrt{x^2+1}}$.

11. Let the relation R in the set $A=\{x\in Z: 0\le x\le 12\}$, given by $R=\{(a,b): |a-b| \text{ is multiple of } 4\}$. Then [1], the equivalence class containing 1, is

(a) $\{1,5,9\}$ (b) $\{0,1,2,5\}$
(c) ϕ (d) A

Sol. Let $(1,x)\in R$ for all $x\in A \Rightarrow |1-x|$ is multiple of 4.
That is, $x=1,5,9$.
Hence, $[1]=\{1,5,9\}$.

12. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is

(a) e^{y-x} (b) e^{x+y}

(c) $-e^{y-x}$ (d) $2e^{x-y}$

Sol. On dividing both the sides by e^{x+y}, we get : $e^{-y} + e^{-x} = 1$

So, $e^{-y}\left(-\frac{dy}{dx}\right) + e^{-x}(-1) = 0$

$\Rightarrow \left(-\frac{dy}{dx}\right) - e^{y-x} = 0$

$\Rightarrow \frac{dy}{dx} = -e^{y-x}$.

13. Given that matrices A and B are of order $3\times n$ and $m\times 5$ respectively, then the order of matrix $C = 5A + 3B$ is

(a) 3×5 and $m = n$ (b) 3×5

(c) 3×3 (d) 5×5

Sol. Order of 5A is $3\times n$ and that of 3B is $m\times 5$.
Also matrices of same order can be added only, that means order of 5A and that of 3B is same.
Clearly, $3 = m,\ n = 5$. That is, order of 5A is 3×5 and same order will be of 3B.
Hence, order of $C = 5A + 3B$ will be 3×5.

14. If $y = 5\cos x - 3\sin x$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $-y$ (b) y

(c) $25y$ (d) $9y$

Sol. $\frac{dy}{dx} = -5\sin x - 3\cos x$

$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -(5\cos x - 3\sin x)$

$\Rightarrow \frac{d^2y}{dx^2} = -y$.

15. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(adj.A)'$ is equal to

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

Sol. Here $adj.A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$ $\therefore (adj.A)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$.

16. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are

(a) $(0, \pm 4)$ (b) $(\pm 4, 0)$

(c) $(\pm 3, 0)$ (d) $(0, \pm 3)$

Sol. On differentiating w.r.t. x both the sides, we get $\frac{2x}{9}+\frac{2y}{16}\times\frac{dy}{dx}=0$ i.e., $\frac{dy}{dx}=-\frac{16x}{9y}$

As the tangents are parallel to y-axis so, $-\frac{16x}{9y}=\frac{1}{0}$ $\Rightarrow 9y=0$ $\Rightarrow y=0$

Put $y=0$ in $\frac{x^2}{9}+\frac{y^2}{16}=1$, we get : $x=\pm 3$.

The required points are $(\pm 3, 0)$.

17. Given that $A=\left[a_{ij}\right]$ is a square matrix of order 3×3 and $|A|=-7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is

(a) 7 (b) –7
(c) 0 (d) 49

Sol. $\sum_{i=1}^{3} a_{i2}A_{i2}=a_{12}A_{12}+a_{22}A_{22}+a_{32}A_{32}=|A|=-7$.

18. If $y=\log(\cos e^x)$, then $\frac{dy}{dx}$ is

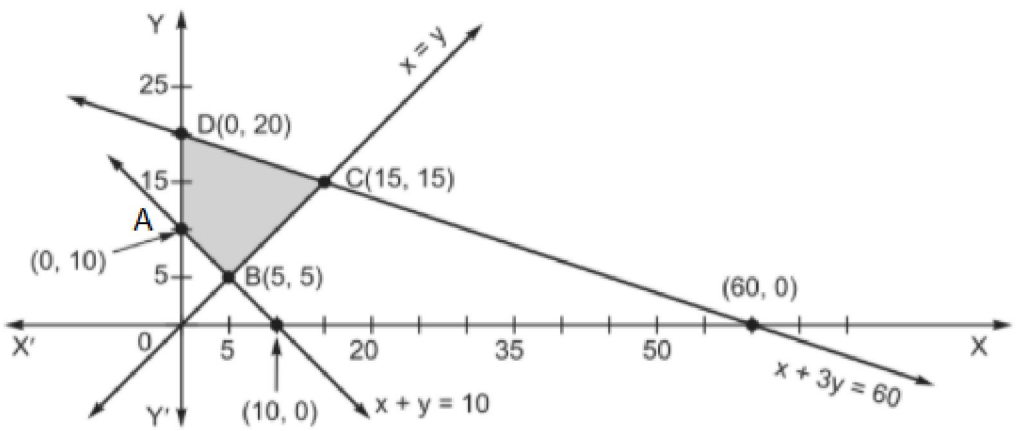
(a) $\cos e^{x-1}$ (b) $e^{-x}\cos e^x$
(c) $e^x \sin e^x$ (d) $-e^x \tan e^x$

Sol. $y=\log(\cos e^x)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{\cos e^x}\times(-\sin e^x)\times e^x$

$\Rightarrow \frac{dy}{dx}=-e^x \tan e^x$.

19. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function $Z=3x+9y$ maximum?



(a) Point B (b) Point C
(c) Point D (d) every point on the line segment CD

Sol. Note that $Z_A=90,\ Z_B=60,\ Z_C=180,\ Z_D=180$.

As Z is maximum at C(15, 15) and D(0, 20) so, maximum value of Z is obtained at all the points of line segment CD.

20. The least value of the function $f(x)=2\cos x+x$ in the closed interval $\left[0,\frac{\pi}{2}\right]$ is

(a) 2 (b) $\frac{\pi}{6}+\sqrt{3}$

(c) $\frac{\pi}{2}$ (d) The least value does not exist

Sol. $\because f'(x) = -2\sin x + 1$

For $f'(x) = -2\sin x + 1 = 0 \quad \Rightarrow \sin x = \frac{1}{2} \quad \Rightarrow x = \frac{\pi}{6} \in \left[0, \frac{\pi}{2}\right]$.

Now $f(0) = 2\cos 0 + 0 = 2$, $f\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{6} + \sqrt{3}$, $f\left(\frac{\pi}{2}\right) = 2\cos\frac{\pi}{2} + \frac{\pi}{2} = \frac{\pi}{2} + 0 = \frac{\pi}{2}$.

So, the least value of $f(x)$ is $\frac{\pi}{2}$.

Section B

Questions in this section carry 1 mark each.

In this section, attempt **any 16 questions** (from **21 - 40**).

21. The function $f: R \to R$ defined as $f(x) = x^3$ is

(a) one-one but not onto (b) not one-one but onto

(c) neither one-one nor onto (d) one-one and onto

Sol. Let $f(x_1) = f(x_2)$ for $x_1, x_2 \in R$.

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

So, f is one-one.

Let $y = x^3$, where $y = f(x)$, $y \in R$.

$\Rightarrow x = y^{1/3}$

That is, every image $y \in R$ has a unique pre-image $x \in R$.

So, f is onto.

22. If $x = a\sec\theta$, $y = b\tan\theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is

(a) $-\frac{3\sqrt{3}\,b}{a^2}$ (b) $-\frac{2\sqrt{3}\,b}{a}$

(c) $-\frac{3\sqrt{3}\,b}{a}$ (d) $-\frac{b}{3\sqrt{3}\,a^2}$

Sol. $\because \frac{dx}{d\theta} = a\sec\theta\tan\theta$, $\frac{dy}{d\theta} = b\sec^2\theta$

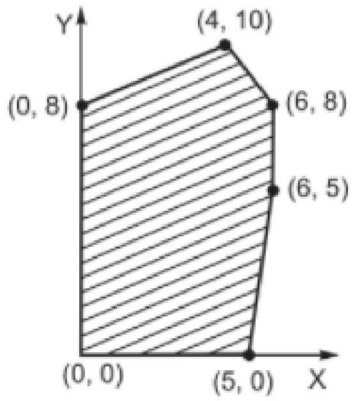
$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = b\sec^2\theta \times \frac{1}{a\sec\theta\tan\theta} = \frac{b}{a} \times \operatorname{cosec}\theta$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a} \times \operatorname{cosec}\theta\cot\theta \times \frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a} \times \operatorname{cosec}\theta\cot\theta \times \frac{1}{a\sec\theta\tan\theta} = -\frac{b}{a^2} \times \cot^3\theta$

Therefore, $\left(\frac{d^2y}{dx^2}\right)_{\text{at } \theta = \frac{\pi}{6}} = -\frac{b}{a^2} \times \cot^3\frac{\pi}{6} = -\frac{b}{a^2} \times \left[\sqrt{3}\right]^3 = -\frac{3\sqrt{3}\,b}{a^2}$.

23. In the given graph, the feasible region for a LPP is shaded.



The objective function $Z = 2x - 3y$, will be minimum at

(a) (4, 10) (b) (6, 8)
(c) (0, 8) (d) (6, 5)

Sol. Here $Z_{(4,10)} = -22,\ Z_{(6,8)} = -12,\ Z_{(0,8)} = -24,\ Z_{(6,5)} = -3$.
Clearly, minimum value of Z is '–24' and it is obtained at (0, 8).

24. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t. $\sin^{-1}x$, $\frac{1}{\sqrt{2}} < x < 1$, is

(a) 2 (b) $\frac{\pi}{2} - 2$
(c) $\frac{\pi}{2}$ (d) –2

Sol. Let $z = \sin^{-1}x \Rightarrow x = \sin z$ also, let $y = \sin^{-1}(2x\sqrt{1-x^2})$.

$\therefore y = \sin^{-1}(2\sin z\sqrt{1-\sin^2 z}) = \sin^{-1}\left[2\sin z|\cos z|\right] = \sin^{-1}\left[2\sin z\cos z\right] = \sin^{-1}\sin 2z$

$\Rightarrow y = \sin^{-1}\sin(\pi - 2z)$

$\Rightarrow y = (\pi - 2z)$

$\therefore \frac{dy}{dz} = -2$.

$\because \frac{1}{\sqrt{2}} < x < 1 \Rightarrow \frac{1}{\sqrt{2}} < \sin z < 1 \Rightarrow \frac{\pi}{4} < z < \frac{\pi}{2} \Rightarrow 0 < \pi - 2z < \frac{\pi}{2}$.

25. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then

(a) $A^{-1} = B$ (b) $A^{-1} = 6B$
(c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$

Sol. Consider $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$

$\Rightarrow \left(\frac{1}{6}A\right)B = I \qquad \therefore B^{-1} = \frac{1}{6}A$.

26. The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing in $(-\infty,-2)$ and strictly decreasing in $(-2,\infty)$
(b) strictly decreasing in $(-2, 3)$
(c) strictly decreasing in $(-\infty,3)$ and strictly increasing in $(3,\infty)$
(d) strictly decreasing in $(-\infty,-2)\cup(3,\infty)$

Sol. $\because f'(x)=6x^2-6x-36=6(x-3)(x+2)$
For $f'(x)=6(x-3)(x+2)=0 \Rightarrow x=-2, 3$
As $f'(x)<0$ for all $x\in(-2,3)$
So, $f(x)$ is strictly decreasing in $x\in(-2,3)$.

27. Simplest form of $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$, $\pi<x<\dfrac{3\pi}{2}$ is

(a) $\dfrac{\pi}{4}-\dfrac{x}{2}$ (b) $\dfrac{3\pi}{2}-\dfrac{x}{2}$

(c) $-\dfrac{x}{2}$ (d) $\pi-\dfrac{x}{2}$

Sol. $$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}\right)$$

$$\Rightarrow \quad =\tan^{-1}\left(\frac{\sqrt{2}\left|\cos\frac{x}{2}\right|+\sqrt{2}\left|\sin\frac{x}{2}\right|}{\sqrt{2}\left|\cos\frac{x}{2}\right|-\sqrt{2}\left|\sin\frac{x}{2}\right|}\right)=\tan^{-1}\left(\frac{-\cos\frac{x}{2}+\sin\frac{x}{2}}{-\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$$

$$\Rightarrow \quad =\tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\Rightarrow \quad =\left(\frac{\pi}{4}-\frac{x}{2}\right).$$

As $\pi<x<\dfrac{3\pi}{2} \Rightarrow \dfrac{\pi}{2}<\dfrac{x}{2}<\dfrac{3\pi}{4} \Rightarrow -\dfrac{\pi}{2}<\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)<-\dfrac{\pi}{4}$.

28. Given that A is a non-singular matrix of order 3 such that $A^2=2A$, then value of $|2A|$ is
(a) 4 (b) 8
(c) 64 (d) 16

Sol. $A^2=2A$
$\Rightarrow |A^2|=|2A|$
$\Rightarrow |A|^2=2^3|A|$
$\Rightarrow |A|=8$ ($\because$ A is non-singular matrix
Now $|2A|=2^3|A|=8\times8=64$.

29. The value of b for which the function $f(x)=x+\cos x+b$ is strictly decreasing over **R** is
(a) $b<1$ (b) No value of b exists
(c) $b\le1$ (d) $b\ge1$

Sol. As $f'(x)=1-\sin x$

Note that $f'(x)>0$ for all $x\in R$.

That means, 'no value of b exists' for which the function f (x) is strictly decreasing over **R**.

30. Let R be the relation in the set N given by $R=\{(a,b):a=b-2,\ b>6\}$, then

(a) $(2,4)\in R$ (b) $(3,8)\in R$

(c) $(6,8)\in R$ (d) $(8,7)\in R$

Sol. Note that, only $(6,8)\in R$.

As $(6,8)$ satisfies $a=b-2,\ b>6$.

31. The point(s), at which the function f given by $f(x)=\begin{cases}\frac{x}{|x|}, & x<0\\ -1, & x\geq 0\end{cases}$ is continuous, is/are

(a) $x\in R$ (b) $x=0$

(c) $x\in R-\{0\}$ (d) $x=-1$ and 1

Sol. $\because f(x)=\begin{cases}\frac{x}{-x}=-1, & x<0\\ -1, & x\geq 0\end{cases}$

That is, $f(x)=-1\ \forall x\in R$.

As f (x) is a constant function so, it is continuous for all $x\in R$.

32. If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$, then the values of k, a and b respectively are

(a) $-6,-12,-18$ (b) $-6,-4,-9$

(c) $-6,4,9$ (d) $-6,12,18$

Sol. As $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ so, $kA=\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}$

So, $\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$

By the def. of equality of matrices, we get $2k=3a,\ 3k=2b,\ -4k=24$.

On solving these equations, we get $k=-6,\ a=-4,\ b=-9$.

33. A linear programming problem is as follows :

Minimize $Z=30x+50y$

Subject to constraints,

$3x+5y\geq 15,$

$2x+3y\leq 18,$

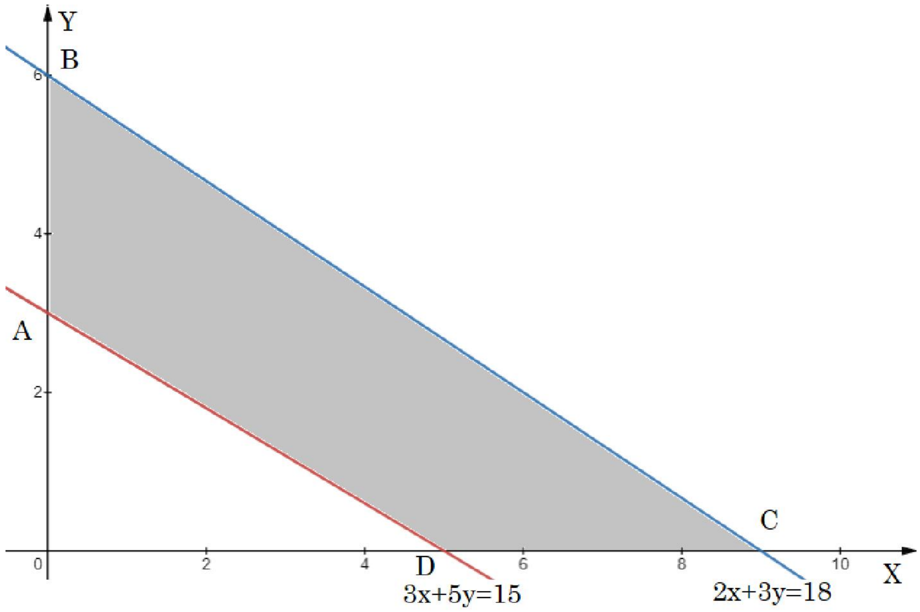
$x\geq 0,\ y\geq 0$

In the feasible region, the minimum value of Z occurs at

(a) a unique point (b) no point

(c) infinitely many points (d) two points only

Sol. Consider the following graph.



Corner Points	Value of Z
A(0, 3)	150
B(0, 6)	300
C(9, 0)	270
D(5, 0)	150

Clearly, the minimum value of Z is 150 and it occurs at A(0, 3) and D(5, 0).
So, this minimum value of Z will occur at all the points (infinitely many points) of line segment joining AD.

34. The area of a trapezium is defined by the function f and given by $f(x)=(10+x)\sqrt{100-x^2}$, then the area when it is maximized is

(a) $75\ \text{cm}^2$ (b) $7\sqrt{3}\ \text{cm}^2$

(c) $75\sqrt{3}\ \text{cm}^2$ (d) $5\ \text{cm}^2$

Sol. Let $u=(10+x)^2(100-x^2)$, where $u=[f(x)]^2$

$\Rightarrow u=(10+x)^3(10-x)$

$\Rightarrow \dfrac{du}{dx}=(10+x)^3(-1)+3(10-x)(10+x)^2=(10+x)^2(20-4x)$

and, $\dfrac{d^2u}{dx^2}=-4(10+x)^2+2(20-4x)(10+x)$

For $\dfrac{du}{dx}=(10+x)^2(20-4x)=0 \Rightarrow x=5$, as $x\neq -10$

$\because \left(\dfrac{d^2u}{dx^2}\right)_{\text{at } x=5}=-4(15)^2+0=-900<0$ so, u is maximum at $x=5$.

Therefore, $f(x)$ is also maximum when $x=5$.

Now maximum area is, $f(5)=(10+5)\sqrt{100-5^2}=75\sqrt{3}\ \text{cm}^2$.

35. If A is square matrix such that $A^2=A$, then $(I+A)^3-7A$ is equal to

(a) A (b) $I+A$

(c) $I-A$ (d) I

Sol. Consider $(I+A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$

$\Rightarrow (I+A)^3 = I + 3IA + 3A + A^2A$ $(\because A^2 = A,\ IA = A$

$\Rightarrow (I+A)^3 = I + 3A + 3A + AA$

$\Rightarrow (I+A)^3 = I + 6A + A$

$\Rightarrow (I+A)^3 = I + 7A$

$\Rightarrow (I+A)^3 - 7A = I$.

36. If $\tan^{-1}x = y$, then

(a) $-1 < y < 1$ (b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

Sol. As $\tan^{-1}x = y$ so, clearly $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

37. Let $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Based on the given information, f is best defined as

(a) surjective function (b) injective function

(c) bijective function (d) function

Sol. As every pre-image $x \in A$ has a unique image $y \in B$.

So, f is injective function.

38. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by

(a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

(c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

Sol. $14A^{-1} = 14 \times \frac{1}{|A|}(\text{adj.}A)$

$\Rightarrow 14A^{-1} = 14 \times \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow 14A^{-1} = 2\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$.

39. The point (s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are

(a) (–2, 19) (b) (2, –9)

(c) (±2,19) (d) (–2, 19) and (2, –9)

Sol. We have $\frac{dy}{dx} = 3x^2 - 11$

As tangent is given as $y = x - 11$ (whose slope is 1) so, $3x^2 - 11 = 1 \quad \Rightarrow x = 2, -2$.

That means, $y = 8 - 22 + 5 = -9,\ y = -8 + 22 + 5 = 19$.

But (–2, 19) does not satisfy the given equation of tangent.

Hence, (2, –9) is the only required point.

40. Given that $A=\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ and $A^2=3I$, then

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$
(c) $3-\alpha^2-\beta\gamma=0$ (d) $3+\alpha^2+\beta\gamma=0$

Sol. $\because A^2=3I$

$\therefore \begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}=3\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}3 & 0 \\ 0 & 3\end{bmatrix}$

On comparing the corresponding terms in both matrices, we get $\alpha^2+\beta\gamma=3$.

That is, $3-\alpha^2-\beta\gamma=0$.

Section C

Questions in this section carry 1 mark each.
In this section, attempt **any 8 questions** (from **41 - 50**).
Questions **46 - 50** are based on **Case-Study**.

41. For an objective function $Z=ax+by$, where $a, b>0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is

(a) $b-3a=0$ (b) $a=3b$
(c) $a+2b=0$ (d) $2a-b=0$

Sol. As $Z_{(30,30)}=Z_{(0,40)}$ so, $30a+30b=0+40b$

$\Rightarrow 3a=b$ i.e., $b-3a=0$.

42. For which value of m, is the line $y=mx+1$ a tangent to the curve $y^2=4x$?

(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 3

Sol. $y^2=4x \ldots$(i) and $y=mx+1 \ldots$(ii)

By (i) and (ii), $(mx+1)^2=4x$

$\Rightarrow m^2x^2+2mx+1=4x$

$\Rightarrow m^2x^2+(2m-4)x+1=0$

As the line (ii) is tangent to the curve (i) so, line will touch the curve at only one point.

Hence we must have $(2m-4)^2-4m^2\times 1=0$

$\Rightarrow -16m+16=0$

$\Rightarrow m=1$

43. The maximum value of $\left[x(x-1)+1\right]^{\frac{1}{3}}$, $0\le x\le 1$ is

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) $\sqrt[3]{\frac{1}{3}}$

Sol. Let $f(x) = \left[x(x-1)+1\right]^{\frac{1}{3}}$, $0 \le x \le 1$

$\Rightarrow f'(x) = \dfrac{2x-1}{3(x^2-x+1)^{2/3}}$

For $f'(x) = \dfrac{2x-1}{3(x^2-x+1)^{2/3}} = 0 \quad \Rightarrow x = \dfrac{1}{2} \in [0,1]$

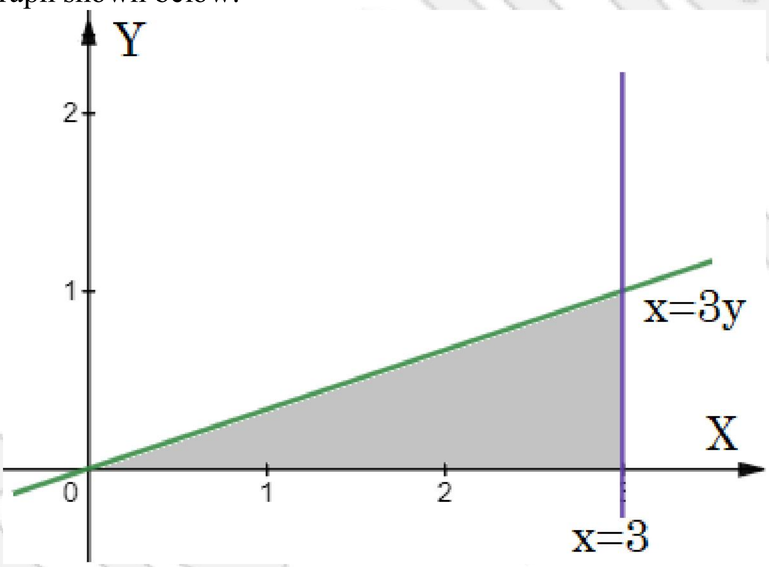
So, $f(0) = \left[0(0-1)+1\right]^{\frac{1}{3}} = 1$, $f\left(\dfrac{1}{2}\right) = \left[\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}-1\right)+1\right]^{\frac{1}{3}} = \left(\dfrac{3}{4}\right)^{1/3}$, $f(1) = \left[1(1-1)+1\right]^{\frac{1}{3}} = 1$.

Hence, the maximum value of $f(x)$ is '1'.

44. In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \ge 0$, $y \ge 0$, $0 \le x \le 3$. The feasible region

(a) is not in the first quadrant
(b) is bounded in the first quadrant
(c) is unbounded in the first quadrant
(d) does not exist

Sol. Consider the graph shown below.



Clearly, the feasible region is bounded in the first quadrant.

45. Let $A = \begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then

(a) $|A| = 0$ (b) $|A| \in (2,\infty)$
(c) $|A| \in (2,4)$ (d) $|A| \in [2,4]$

Sol. Consider $|A| = \begin{vmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{vmatrix}$

On expanding along R_1, we get $|A| = 2 + 2\sin^2\alpha$.

As $-1 \le \sin\alpha \le 1$ for all $\alpha \in [0, 2\pi]$ so, $0 \le \sin^2\alpha \le 1$

$\Rightarrow 0 \le 2\sin^2\alpha \le 2$

$\Rightarrow 2 \le 2\sin^2\alpha + 2 \le 4$

$\Rightarrow 2 \le |A| \le 4$ i.e., $|A| \in [2,4]$.

CASE STUDY

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs ₹ 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to ₹ 1200 per hour. Assume the speed of the train as v km/h.

Based on the given information, answer the following questions.

46. Given that the fuel cost per hour is k times the square of the speed the train generates in km/h, the value of k is

(a) $\frac{16}{3}$ (b) $\frac{1}{3}$

(c) 3 (d) $\frac{3}{16}$

Sol. $\text{Fuel cost} = k(\text{speed})^2$

$\Rightarrow 48 = k \times (16)^2 \qquad \Rightarrow k = \frac{3}{16}$

47. If the train has travelled a distance of 500 km, then the total cost of running the train is given by function

(a) $\frac{15}{16}v + \frac{600000}{v}$ (b) $\frac{375}{4}v + \frac{600000}{v}$

(c) $\frac{5}{16}v^2 + \frac{150000}{v}$ (d) $\frac{3}{16}v + \frac{6000}{v}$

Sol. Total cost of running the train (let C) $= \frac{3}{16}v^2 t + 1200t$

As the distance covered by train is 500 km so, $t = \frac{500}{v}$

$\therefore C = \frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$

$\Rightarrow C = \frac{375}{4}v + \frac{600000}{v}$.

48. The most economical speed to run the train is

(a) 18 km/h (b) 5 km/h

(c) 80 km/h (d) 40 km/h

Sol. Note that $\frac{dC}{dv}=\frac{375}{4}-\frac{600000}{v^2}$ and $\frac{d^2C}{dv^2}=\frac{1200000}{v^3}$

For $\frac{dC}{dv}=\frac{375}{4}-\frac{600000}{v^2}=0$, $v^2=6400$ i.e., $v=80$ km/h

$\because \left(\frac{d^2C}{dv^2}\right)_{\text{at } v=80}=\frac{1200000}{(80)^3}>0$ so, C will be minimum when $v=80$ km/h.

49. The fuel cost for the train to travel 500 km at the most economical speed is

(a) ₹ 3750 (b) ₹ 750

(c) ₹ 7500 (d) ₹ 75000

Sol. Fuel cost for running 500 km, $\frac{375}{4}v=\frac{375}{4}\times 80=7500$ (in ₹).

50. The total cost of the train to travel 500 km at the most economical speed is

(a) ₹ 3750 (b) ₹ 75000

(c) ₹ 7500 (d) ₹ 15000

Sol. Total cost for running 500 km, $\frac{375}{4}v+\frac{600000}{v}=\frac{375}{4}\times 80+\frac{600000}{80}=15000$ (in ₹).

ANSWER KEY

01.	(d)	**02.**	(b)	**03.**	(d)	**04.**	(c)	**05.**	(b)	**06.**	(d)	**07.**	(b)
08.	(a)	**09.**	(a)	**10.**	(d)	**11.**	(a)	**12.**	(c)	**13.**	(b)	**14.**	(a)
15.	(c)	**16.**	(c)	**17.**	(b)	**18.**	(d)	**19.**	(d)	**20.**	(c)	**21.**	(d)
22.	(a)	**23.**	(c)	**24.**	(d)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(c)
29.	(b)	**30.**	(c)	**31.**	(a)	**32.**	(b)	**33.**	(c)	**34.**	(c)	**35.**	(d)
36.	(c)	**37.**	(b)	**38.**	(b)	**39.**	(b)	**40.**	(c)	**41.**	(a)	**42.**	(b)
43.	(c)	**44.**	(b)	**45.**	(d)	**46.**	(d)	**47.**	(b)	**48.**	(c)	**49.**	(c)
50.	(d)												

This sample paper has been issued by **CBSE** for **Term 1 (2021-22) Board Exams of class 12 Mathematics (041)**.

Note : We've **re-typed** the official sample paper and, also done the necessary corrections at some places.

If you notice any error which could have gone un-noticed, please do inform us via **WhatsApp @ +919650350480 (message only)** or, via **Email** at **iMathematicia@gmail.com**

Let's learn **Math** with **smile**:-)

- O.P. GUPTA, Math Mentor

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